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ENERGY ANALYSIS OF OPEN REGIONS OF TURBULENT FLOWS — MEAN EDDY ENERGETICS OF A NUMERICAL OCEAN CIRCULATION EXPERIMENT

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ABSTRACT

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The concepts involved in the interpretation of energy budgets in subregions of a turbulent flow are examined in order to determine the processes responsible for the production, transport, and dissipation of energy throughout a dynamically inhomogeneous circulation. An interpretation of the effects of Reynolds stress—mean flow interaction work for open regions is presented in terms of the change in the total mean kinetic energy. In an arbitrary volume of fluid the changes in kinetic energy of the mean flow and the mean kinetic energy of the eddy flow are not generally equal and opposite, so this process is not generally responsible for a conversion of energy between the two forms. These ideas are then applied to a regional kinetic energy analysis of the mesoscale resolution general ocean circulation numerical experiment of Robinson et al. (1977). The spatial structure of the various terms in the equation for the mean eddy kinetic energy is examined. The issues involved in selection of a set of analysis regions are discussed and explored via examination of budgets over different subregions of this flow. Thereby a relatively simple picture of the regional energetics emerges. Mean eddy kinetic energy is produced by conversion of kinetic energy of the mean flow in the net over the recirculation and near field of the northern boundary current system and roughly half of this energy is lost to each of mean eddy pressure work transport and diffusion work. Budgets over subregions of this net source region are much more complex. The interior eddy field is driven by pressure work influx, while the southwestern region has eddy buoyancy work conversion of mean potential energy as its energy source. At every depth level the eddy field draws its kinetic energy from the mean flow, when averaged over the horizontal extent of the basin or over the recirculation and near field.

1. INTRODUCTION

Spatial inhomogeneity of time mean statistical properties is characteristic of many geophysical flows currently under scientific study. Mesoscale resolution numerical general ocean circulation experiments (e.g., Holland and Lin, 1975; Robinson et al., 1977; Semtner and Mintz, 1977), western North Atlantic ocean data (e.g., Brooks and Niiler, 1977; Dantzler, 1977; Luyten,

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1977; Schmitz, 1977) and mid-latitude winter atmospheric data (e.g., Blackmon et al., 1977) have all been found to exhibit inhomogeneous first and second moments. It is necessary therefore to develop simple analysis methods that will give insight into the dynamical effects of, and the processes responsible for, these flow statistics.

Because of the complexity of the space and time variability of such geophysical flows, useful analysis methods have often involved considerable averaging in time and/or space, e.g., seasonal and zonal averages for atmospheric data and long-time and full basin averages for numerical ocean experiments. Insight gained from the results of using these spatially averaged methods has contributed to the mechanistic understanding that exists of the atmospheric and model oceanic general circulations. However, such methods are always susceptible to the problem of producing spatially averaged budgets that may not be characteristic of the dynamical behavior of any actual subregion of the flow itself. Furthermore, they cannot yield information about the effects or causes of the spatially unaveraged structure. Particularly in the turbulent flow of numerical experiments, where every necessary variable is known, analysis methods which enable these issues to be studied should be employed.

The problem of understanding the effects of spatial inhomogeneity of a turbulent flow is here considered via examination of mean turbulent kinetic energy budgets over arbitrary open regions, i.e., with flow across the regional boundary. Interpretation of open regional energy budgets is complicated by the fact that the interaction of Reynolds stresses with the mean flow does not generally produce conversion of energy between the mean turbulent kinetic energy and the kinetic energy of the mean flow, as it does in a closed region. Indeed, this interaction is generally responsible for a net change in the sum of these two kinetic energies rather than simply a local transformation of energy from one type to the other. However, regional integrals of the terms in the mean turbulent kinetic energy budget evaluated over properly selected regions, combined with appropriate understanding of the processes represented by each term, can enable useful interpretation of the regional energy budget to be given.

These ideas are applied to data from the highly time dependent flow of the mesoscale resolution numerical general ocean circulation experiment of Robinson, Harrison, Mintz and Semtner (1977) (henceforth RHMS) in order to investigate the mechanisms responsible for the production, redistribution and dissipation of the mean kinetic energy of its eddy field. Unsmoothed maps of the various energy terms are presented, appropriate analysis regions are discussed and defined and regional budgets are examined. The method is generalizable to many fluid systems and is equally applicable to the study of actual data as to model data.

In Section 2 the analysis method and equations appropriate to RHMS are given. The spatial distribution of the model energy terms is qualitatively and quantitatively examined in Section 3. Energy budgets over various regions are described in Section 4. These results and some implications are discussed in Section 5.

2. OPEN REGIONAL ENERGY ANALYSIS

The problem of open region energy analysis is discussed here in terms of the primitive equation fluid system used in the numerical experiment of RHMS, although the basic ideas are more general.

A. *Energy equations for a turbulent primitive equation fluid*

We examine the mean energetic properties of our turbulent fluid system by introducing an averaging operator, $\bar{\ }$, and decomposing each field into its average (mean field) and a departure from this average (eddy field), $\phi' = \phi - \overline{\phi}$, in the classical Reynolds sense (e.g., Monin and Yaglom, 1971). However, for the numerical analysis the overbar represents a time average over the duration of the data timeseries.

The mean kinetic energy (MKE), $\frac{1}{2}(\overline{u^2 + v^2})$, is the sum of the kinetic energy of the mean flow (KEM), $\frac{1}{2}(\overline{u}^2 + \overline{v}^2)$, and of the mean kinetic energy of the eddy flow (MKEF), $\frac{1}{2}(\overline{u'^2 + v'^2})$. The needed energy equations for this study consist of those for the KEM, MKEF and mean total potential energy:

$$\overline{u}_\lambda \overline{\frac{\partial}{\partial t} u_\lambda} + \frac{\partial}{\partial x_j}\left[\tfrac{1}{2}\overline{u}_j\overline{u}_\lambda\overline{u}_\lambda + \overline{u}_j\overline{\tilde{p}}\right] + \overline{u}_\lambda \frac{\partial}{\partial x_j}\overline{u_j' u_\lambda'} =$$

$$+ A_M\overline{u}_\lambda \frac{\partial^2 \overline{u}_\lambda}{\partial x_\delta \partial x_\delta} + K\overline{u}_\lambda \frac{\partial^2 \overline{u}_\lambda}{\partial z^2} + \alpha g \overline{w}\overline{T} \tag{1a}$$

$$\frac{1}{2}\overline{\frac{\partial}{\partial t}(u_\lambda' u_\lambda')} + \frac{\partial}{\partial x_j}\left[\tfrac{1}{2}\overline{u}_j\overline{u_\lambda' u_\lambda'} + \tfrac{1}{2}\overline{u_j' u_\lambda' u_\lambda'} + \overline{u_j'\tilde{p}'}\right] + \overline{u_\lambda' u_j'}\frac{\partial \overline{u}_\lambda}{\partial x_j} =$$

$$+ A_M\overline{u_\lambda' \frac{\partial^2 u_\lambda'}{\partial x_\delta \partial x_\delta}} + K\overline{u_\lambda' \frac{\partial^2 u_\lambda'}{\partial z^2}} + \alpha g\overline{w' T'} \tag{1b}$$

$$-\alpha g\left[\overline{\frac{\partial}{\partial t} zT} + \frac{\partial}{\partial x_j}\left(\overline{u}_j\overline{T}z + \overline{u_j' T'}z\right)\right] =$$

$$-\alpha g\left[\overline{w}\overline{T} + \overline{w' T'}\right] - \alpha g\left[A_H \frac{\partial^2 \overline{T}z}{\partial x_\delta \partial x_\delta} + Kz\frac{\partial^2 \overline{T}}{\partial z^2}\right] \tag{1c}$$

where Cartesian tensor notation with summation convention has been used, $\delta, \lambda = 1, 2$; $j = 1, 2, 3$; α is from the model equation of state, $\rho = \rho_0[1 - \alpha(T - T_0)]$; T is the effective temperature; $\tilde{p}$ is a dynamic pressure in which the hydrostatic pressure field has been removed; u_λ are the conventional horizontal components of the velocity vector u_j; $w(u_3)$ is the vertical velocity; z is the depth; g is the gravitational acceleration and K, A_H, A_M are constants.

The KEM equation is formed by taking the scalar product of the mean velocity with the vector mean momentum equation. The MKEF equation follows by subtracting the KEM equation from that of the MKE (the mean of the scalar product of the velocity and vector momentum equation) and making use of the identity:

$$\frac{\partial}{\partial x_j}\left(\overline{u'_\lambda u'_j}\,\overline{u}_\lambda\right) = \overline{u'_\lambda u'_j}\,\frac{\partial \overline{u}_\lambda}{\partial x_j} + \overline{u}_\lambda\,\frac{\partial}{\partial x_j}\,\overline{u'_\lambda u'_j} \tag{1d}$$

to obtain the conventional form (Goldstein, 1965; Monin and Yaglom, 1971). The mean total potential energy equation is formed directly from the model heat equation by multiplication by $-\alpha gz$. *

It can be seen from eqs. 1 that the time change ** of a particular energy results from three classes of processes, transport (or flux) processes, diffusive processes and interaction processes. The divergence terms represent transport processes. The diffusive terms represent processes that are not explicitly resolved by the numerical calculation. They provide a dissipative mechanism and allow external stresses and heat fluxes to drive the system. The remaining terms we call interaction processes, since they involve a direct or indirect coupling of two different types of energy as will be seen in the following section.

B. Interaction processes and the concept of conversion

Possible ambiguity associated with alternative forms of the interaction terms in eqs. 1, (e.g., by use of eq. 1d, Lettau, 1954) must be resolved by defining the specific physical processes responsible for the interactions (Lorenz, 1955). In the special case that an interaction process is represented by the same term with opposite sign in two energy equations, the interaction is regarded as a conversion between those two forms of energy. The two interaction processes of eqs. 1 are buoyancy work, defined by the interaction of the total flow with the gravitational field and Reynolds stress—mean flow interaction work, defined by the interaction of the mean flow with the Reynolds stress field.

In these terms buoyancy work is a conversion process responsible for energy exchange between KEM and mean potential energy and between MKEF and potential energy because $\alpha g\overline{w}\overline{T}$ and $\alpha g\overline{w'T'}$ appear with opposite signs in the pairs of eqs. 1a, 1c and 1b, 1c, respectively.

To discuss the Reynolds stress—mean flow interaction work (hereafter interaction work) process, we now consider the energy changes due to interac-

* It can be shown that this equation is the sum of the potential and internal energy equations, simplified by scale analysis (Harrison, 1977), thus it does describe the total potential energy.

** It is convenient to speak of the change of a regional energy value due to a particular process even though all energy integrals are time-invariant by definition of the averaging operator. No confusion will result from this usage so long as it is understood that by the "increase (decrease)" due to a process we mean that positive (negative) work is being done by the particular process on the volume of fluid. The sum of changes due to all possible processes will necessarily be zero for a flow in statistical equilibrium.

tion work. Over a volume of fluid the total work done on that volume of fluid is given by:

$$\rho_0 \int_V \frac{\partial}{\partial x_j} (\overline{u_j' u_\lambda'}\, \overline{u}_\lambda)\, \mathrm{d}V$$

and is responsible for a change in the integrated total mean energy (MKE). From eqs. 1a, b, d it is seen that this MKE change is the sum of interaction work changes in KEM and MKEF described by:

$$\rho_0 \int_V \overline{u}_\lambda \frac{\partial}{\partial x_j} \overline{u_\lambda' u_j'}\, \mathrm{d}V \qquad \text{and} \qquad \rho_0 \int \overline{u_\lambda' u_j'} \frac{\partial \overline{u}_\lambda}{\partial x_j}\, \mathrm{d}V$$

respectively. For this reason we define:

$$\frac{\partial}{\partial x_j} \overline{u_\lambda' u_j'}\, \overline{u}_\lambda\ , \quad \overline{u}_\lambda \frac{\partial}{\partial x_j} \overline{u_\lambda' u_j'}\ , \quad \text{and} \quad \overline{u_\lambda' u_j'} \frac{\partial \overline{u}_\lambda}{\partial x_j}$$

to be the MKE, KEM, and MKEF interaction work terms, respectively. These labels are suggested by an analogy between viscous and Reynolds stresses. When the viscous stress tensor interaction with a laminar flow is examined, a relationship formally analogous to (1d) exists with the replacement $\overline{u_\lambda' u_j'} \rightleftharpoons \sigma_{ij}$, KEM $\rightleftharpoons$ kinetic energy, and MKEF $\rightleftharpoons$ internal energy are made (Townsend, 1956, §2.7). It must be emphasized that this formal correspondence does not imply a similar physical correspondence, for the properties of the viscous and Reynolds stress tensors can be quite different.

If integrated over a mechanically closed volume of fluid (e.g., no normal boundary velocity) then (1d) shows that the integrals of the KEM and MKEF interaction terms are equal and opposite. Thus over such a region interaction work is a conversion process. Closed domain global energetic balances of course utilize this result.

However, if integration over an arbitrary volume of fluid is considered, (1d)

TABLE I

Types of regions for MKEF regional analysis

Region type	Properties
1	Interaction work acts roughly like a regional conversion process: $\int_R \overline{u_\lambda' u_j'} \frac{\partial \overline{u}_\lambda}{\partial x_j}\, \mathrm{d}V \cong -\int_R \overline{u}_\lambda \frac{\partial}{\partial x_j} \overline{u_j' u_\lambda'}\, \mathrm{d}V$
2	Interaction work does not act like a regional conversion process, but can be neglected compared to other work terms
3	Interaction work does not act like a regional conversion process, and cannot be neglected compared to other work terms

illustrates that the KEM and MKEF interaction work terms will generally not be equal and opposite and so interaction work does not generally act as a conversion process.

To understand the regional effects of interaction work, it is convenient to distinguish between the three possible results shown in Table I when the interaction terms are integrated over an open region. If the KEM and MKEF interaction work are approximately equal and opposite in the net over the region, then the effect of interaction work is to act as a regional conversion process and such a region will hereafter be classified Type 1. Should the regional integrals not be roughly equal and opposite then the region is classified either Type 2, if interaction work is negligible in the regional budget compared to the other transport, conversion and diffusion terms or Type 3, if interaction work cannot be neglected. The physical importance of these distinctions appears when trying to define useful analysis regions.

C. Terms for the turbulent kinetic energy balance

To determine the means by which MKEF is produced, transported around within the basin and affected by diffusive processes, the terms in eqs. 1b and 1d and their volume integrals over various open regions, R, are examined and compared. The regionally integrated terms that will be discussed and the name here adopted for the physical process associated with each are:

$\left\{\dfrac{\partial}{\partial x_j}\left(\frac{1}{2}\overline{u}_j\,\overline{u'_\lambda u'_\lambda}\right)\right\}_R$	boundary transport of MKEF due to mean flow advection of MKEF
$\left\{\dfrac{\partial}{\partial x_j}\left(\frac{1}{2}\overline{u'_j u'_\lambda u'_\lambda}\right)\right\}_R$	boundary transport of MKEF due to mean eddy advection of eddy energy
$\left\{\dfrac{\partial}{\partial x_j}\overline{u'_j\tilde{p}'}\right\}_R$	boundary transport of MKEF due to boundary mean eddy pressure work
$\left\{A_M\overline{u'_\lambda\dfrac{\partial^2 u'_\lambda}{\partial x_\delta\partial x_\delta}}\right\}_R$	mean eddy lateral diffusion work (dissipation and regional boundary stress work)
$\left\{K\overline{u'_\lambda\dfrac{\partial^2 u'_\lambda}{\partial z^2}}\right\}_R$	mean eddy vertical diffusion work
$\left\{\alpha g\overline{w'T'}\right\}_R$	mean eddy buoyancy work conversion between MKEF and potential energy
$\left\{\overline{u'_\lambda u'_j}\dfrac{\partial\overline{u}_\lambda}{\partial x_j}\right\}_R$	MKEF Reynolds stress—mean flow interaction work
$\left\{\overline{u}_\lambda\dfrac{\partial}{\partial x_j}\overline{u'_\lambda u'_j}\right\}_R$	KEM Reynolds stress—mean flow interaction work

$\left\{\frac{\partial}{\partial x_j}(\overline{u}_\lambda \overline{u'_\lambda u'_j})\right\}_R$ MKE Reynolds stress—mean flow interaction work

$\left\{\overline{\frac{\partial}{\partial t}(\frac{1}{2}u'_\lambda u'_\lambda)}\right\}_R$ storage of MKEF over the averaging interval (= 0 if statistics are stationary in time)

where

$$\{\phi\}_R \equiv \rho_0 \int_R \phi \mathrm{d}(\mathrm{Vol})_R$$

D. Selection of analysis regions

In a spatially inhomogeneous flow many different factors can affect the selection of appropriate analysis regions. It is physically desirable that the analysis region be a quasi-homogeneous part of the flow in order to avoid averaging over areas of different behavior. However, the chosen type of homogeneity can be defined using the kinematical characteristics of the flow, the dynamical balance of terms in the flow equations or the distribution of the energetic processes themselves. Generally speaking each of the three criteria will yield different analysis regions.

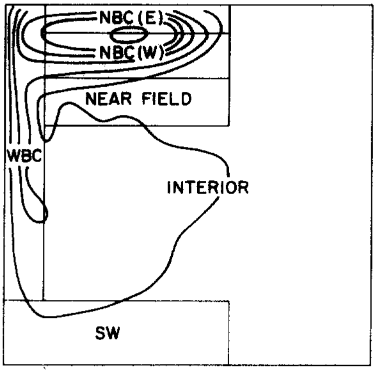
The ideal set of energy analysis regions would be one in which each region is kinematically and dynamically distinct, yet spatially homogeneous within the regional boundaries, and also of Type 1 or Type 2 (Section 2.B). When kinematical or balance of terms homogeneity leads to regions of Type 3 there will be limitations on the physical conclusions that can be drawn from such budgets because interaction work does not act as conversion process. Conversely budgets over regions of Type 1 where the flow cannot be regarded as kinematically or dynamically homogeneous may be difficult to interpret physically.

We have found it necessary to examine the unsmoothed spatial distribution of energy terms, to compute integrals of these terms over small horizontal areas of different vertical extent, and to combine these small regional integrals in various ways dictated by the above selection criteria in order to ultimately arrive at a satisfactory set of analysis regions that yield a relatively simple energetic description of the flow.

3. RHMS MKEF ENERGY TERMS

Plots of the energy terms of (1b,d), along with regional integrals over small volumes of the basin, are displayed and/or described in this section. The small regional integrals serve the important role of reducing the large amount of spatial information to a quantity that can be easily assimilated. It is convenient for descriptive purposes to introduce the regions of the flow of RHMS that were found to be kinematically or dynamically distinct through analysis of

(a)



(b)

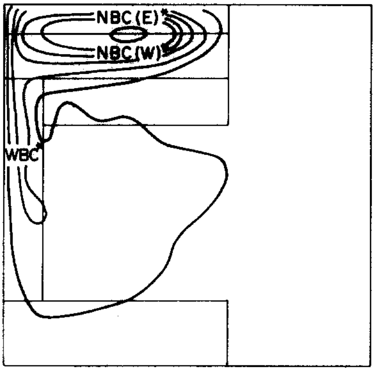


Fig. 1. The mean transport streamfunction field of Robinson et al. (1977) with (a) kinematically or dynamically distinct regions of the flow outlined and labeled, and (b) alternative kinematically distinct boundary current regions. Contour interval is $60 \cdot 10^6\ m^3\ s^{-1}$ relative to the boundary.

the flow characteristics and heat, momentum and vorticity balances. These regions are shown and labeled in Fig. 1a, superimposed upon the mean transport streamfunction, and will be used in descriptions of the geographical variation of energy terms. Shown in Fig. 1b are alternate boundary current regions that will be useful in Section 4.

The extent to which the small scale features of some of the plots should be believed deserves comment. Whenever convenient, energy term variances have been computed in order to do simple tests of significance. Using the RHMS test of significance leads to the conclusion that maximum amplitude point values are generally reliably different from zero at the 99% level. The significance of smaller values varies from energy term to term and can be marginal or even zero. A thorough description of significance for the various terms is felt to be unnecessary for our qualitative purposes. In addition to questions of significance it is necessary in some regions to consider the numerical reliability of the original data as suspect (see RHMS), and these areas will be pointed out as appropriate. Unless otherwise noted, variation of the energy terms on the scale of the mean flow in intense current regions and the eddy scale elsewhere is felt to be plausible. It is not known whether the smaller scale features are indeed physically characteristic of intense current regions or should be regarded as numerical artifacts.

In order to insure proper representation of the finite difference energetics of model subregions, the various terms in the above equations and their integrals have generally been evaluated in strict adherence to the numerical model finite difference energy equation expressions. Only in Section 3.C. 3 was it so much more convenient to abandon the proper finite difference form that another form was used and the limited consequences of this modification are considered there.

The use of the strict staggered grid finite difference energetics forms introduces one non-trivial constraint into this analysis because buoyancy work and vertical integrals of buoyancy work are defined over different grids than are the other KEM and MKEF terms (Haney, 1971). Because of this, energy budgets are not defined for individual grid boxes. Budgets can be rigorously defined only if the regions extend over the full depth of the basin. It would be necessary to introduce interpolations and approximations to the finite difference energetics in order to obtain balanced budgets over regions of limited vertical extent.

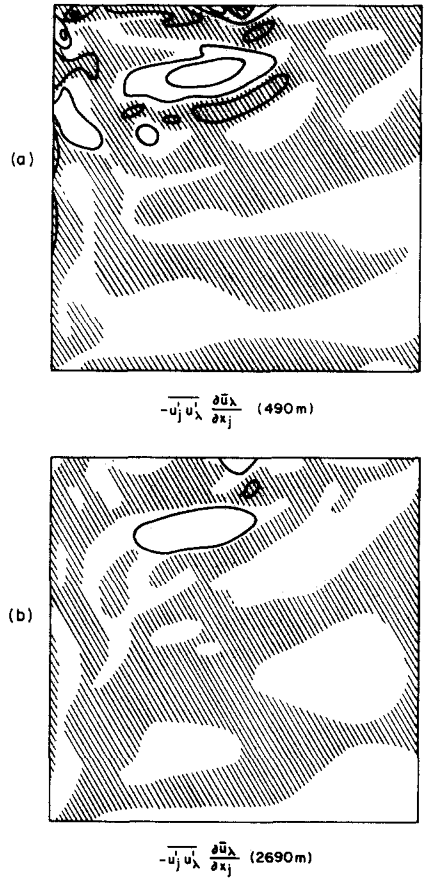
A. Interaction work processes

1. Reynolds stress—mean flow interaction work

The KEM and MKEF Reynolds stress—mean flow interaction terms are hereafter defined with the following signs included: $\overline{u}_\lambda\, \partial\, \overline{u_j' u_\lambda'}/\partial x_j$ for KEM and $-\overline{u_\lambda' u_j'}\, \partial \overline{u}_\lambda/\partial x_j$ for MKEF. This sign convention is introduced so that positive values of the MKEF (KEM) interaction term indicate an increase (decrease) of MKEF (KEM) due to interaction work processes. Thus, whenever values are positive for both fields, interaction work processes decrease KEM and at the same time increase MKEF over that region of fluid. Whenever values are of the same sign and magnitude, conversion (2.B) between KEM and MKEF is taking place.

Consider briefly point values of the MKEF term at different levels in the model grid. Fig. 2 shows plots of $-\overline{u_j' u_\lambda'}\, \partial \overline{u}_\lambda/\partial x_j$ at upper thermocline (490 m) and deep water (2690 m) levels. Stippled regions indicate negative values and the first contour line is drawn for magnitudes of 10^{-5} erg g^{-1} s^{-1}. Other lines correspond to factors of ten increase in magnitude so that the next contour corresponds to 10^{-4}, etc. The upper thermocline plot is generally typical of the behavior of the upper four levels in the numerical model.

The MKEF interaction term plots (Fig. 2) are spatially variable on the eddy scale away from intense currents and are variable on the boundary current scale in intense current regions. Within the NBC(W) and Near Field



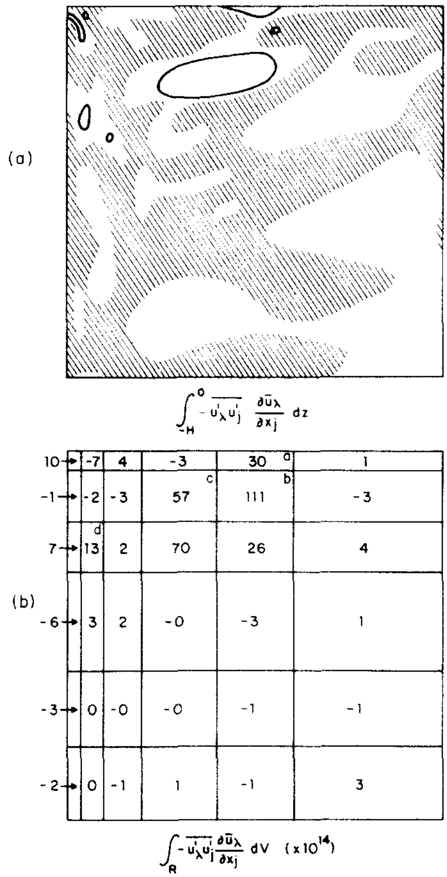


Fig. 2. MKEF Reynolds stress—mean flow interaction term in (a) the upper thermocline (490 m), and (b) the deep water (2690 m). Stippled regions indicate negative values and the first contour line corresponds to a magnitude of 10^{-5} erg g^{-1} s^{-1}, while each further contour line corresponds to an increase in magnitude of a factor of ten. Positive values indicate that MKEF is being increased by interaction processes.

Fig. 3. MKEF Reynolds stress—mean flow interaction term. (a) Vertically integrated field. Plotting convention is as in Fig. 2 except the first contour line now corresponds to 10^{-1} erg cm g^{-1} s^{-1}. (b) Subregional integrals. Units are 10^{-14} erg s^{-1}.

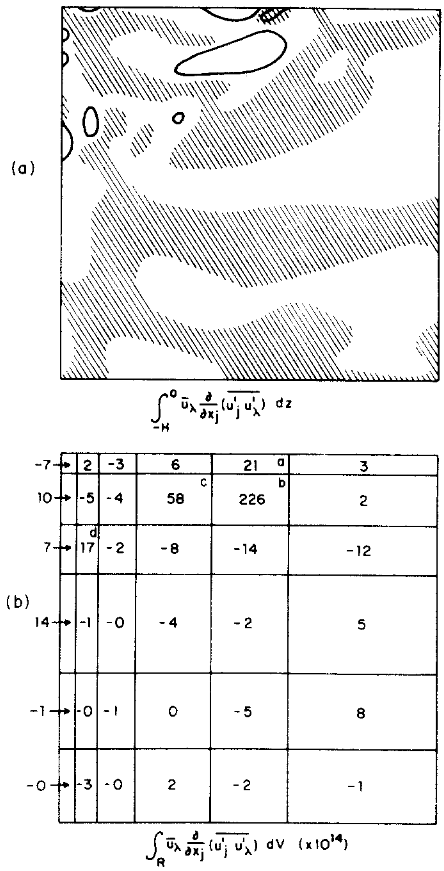
regions point values are generally of the same sign throughout the water column, but elsewhere there is sign variation between the two depths. The only location where large point values ($>10^{-4}$ erg g^{-1} s^{-1}) are found systematically over many grid points is in the shallower depths of an area overlapping the NBC(W) and Near Field regions, and the values are positive. There is considerable large amplitude and small space scale, O (50 km), variation in the northern part of the WBC and along the northern wall in the NBC(E). This variation, as well as the spatial structure described above, cannot be attributed to inadequacy of the statistics, but is either characteristic of the physical model intense current regions or of numerical model inadequacies within these regions.

Vertical integration of the MKEF interaction term results in Fig. 3a, where the plotting conventions are the same as for Fig. 2 except that the first heavy line now corresponds to values of 10^{-1} erg cm g^{-1} s^{-1}. Except in part of the NBC(E) Fig. 3a resembles Fig. 2a, and the same area of the NBC(W) and Near Field remains the only significant area of large positive values. There are no regions of similarly large negative values, where energy is lost from MKEF. Subregional volume integrals are shown in Fig. 3b and confirm the impressions given by Fig. 3a. The central and eastern subregions of the NBC(W) and Near Field and the eastern subregion of the NBC(E) present the largest integral values, and indicate an increase in MKEF due to interaction work.

Fig. 4 presents the KEM interaction term with the same conventions as Fig. 3. Although not shown, the four upper level KEM interaction term plots tend to resemble Fig. 4a, with the deep level plot similar in the NBC(W), but generally of opposite sign elsewhere. The dominant feature of Fig. 4a, b is the relatively large area of the NBC(W) where there is loss of energy from KEM due to interaction work. Nowhere is there correspondingly great gain of energy to KEM. From Fig. 4b, it is seen that the combined energy loss in the eastern and central NBC(W) subregions is ten times greater than the gain or loss in any other subregion and typically thirty or more times greater than that in subregions away from the boundaries.

In order to begin discussion of the energy conversion properties of interaction work in this flow, consider Fig. 5. The plot presented in Fig. 5a shows those areas of the flow in which there is pointwise decrease of KEM and increase of MKEF (case *a*), pointwise increase of KEM and decrease of MKEF (case *b*) and simultaneous pointwise increase or decrease of KEM and MKEF (case *c*). There is relatively limited area of case *a*, somewhat greater area (~20% of the basin) of case *b* and so the dominant behavior is of case *c*. Subregional integrals are presented in Fig. 5b, with a " + " used to denote regions whose behavior is of case *a*, " — " for regions of case *b* and regions of case *c* are left blank. Only in subregions marked " + " is it possible for there to be conversion of KEM into MKEF, but examination of the values of the subregional integrals given in Figs. 3b and 4b reveals that only the central NBC(W) subregion is of Type 1 (see Section 2.B) and has significant magnitude. The other NBC(W) and Near Field subregions with large values do not show Type 1 behavior.

In order to give the distribution of interaction work with depth over subregions with vertically integrated case *a* behavior, we present Table II. The first data column gives the MKEF/KEM integral over the horizontal extent of the basin and the depth band indicated. The other columns present integrals over the subregions labeled with letters in Figs. 3b and 4b and the various depth bands. Integrated over the basin it is seen that each level constitutes a Type 1 region in which KEM is converted into MKEF. Within the



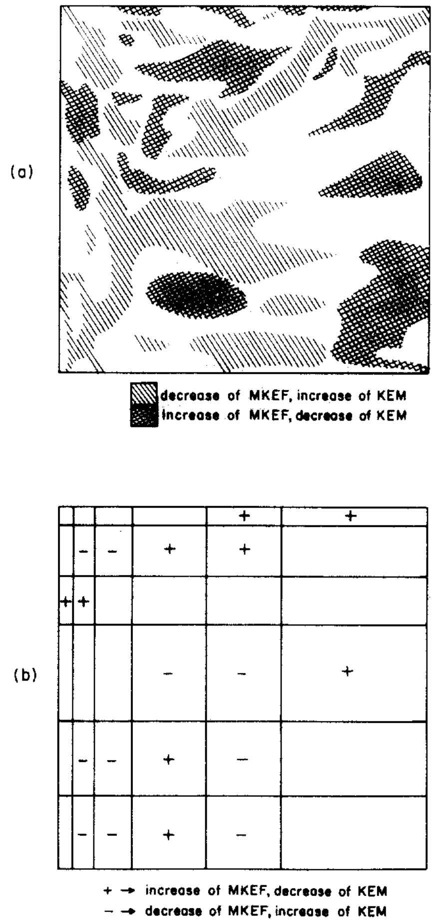


Fig. 4. KEM Reynolds stress—mean flow interaction term as in Fig. 3 except that positive values indicate that KEM is being decreased by interaction processes. Note that maximum positive values of KEM and MKEF are found in different areas (compare with Fig. 3).

Fig. 5. Comparison of KEM and MKEF interaction terms. (a) Comparison of sign of vertically integrated terms. Heavy (light) stippling indicates a decrease of KEM (MKEF) and an increase of MKEF (KEM), while no stippling indicates a simultaneous increase or decrease of both KEM and MKEF. (b) Subregional integral comparison of sign of energy changes.

NBC subregions there is relatively little variation with depth of the sign and magnitude of each interaction work, while the WBC subregions are largest in the upper three depth bands. Except in NBC(W)2, there are few subregions of Type 1. A maximum volume normalized value is about 10^{-4} erg cm^{-3} s^{-1} and a typical value is $O\,(10^{-5})$ at each depth.

2. Eddy buoyancy work

The mean eddy buoyancy work, $\alpha g \overline{w'T'}$, at 325 m is shown in Fig. 6a and

TABLE II

Regional interaction work integrals vs. depth (see text)

$$\left(\int_R -\overline{u'_\lambda u'_j}\,\frac{\partial \bar{u}_\lambda}{\partial x_j}\,\mathrm{d}V\right)\Big/\left(\int_R \bar{u}_\lambda\,\frac{\partial}{\partial x_j}\,\overline{u'_\lambda u'_j}\,\mathrm{d}V\right)(\times 10^{14}\ \mathrm{erg\ s^{-1}})$$

Depths (m)	Region				
	Basin	NBC(E)[a]	NBC(W)1[b]	NBC(W)2[c]	WBC[d]
0—100	33/34	7/1	17/33	9/8	4/5
100—325	49/49	8/5	25/45	14/10	6/7
325—900	58/58	6/6	25/53	15/15	3/4
900—2000	52/50	6/4	20/44	9/12	0/1
2000—4000	71/66	3/5	24/51	10/11	—1/0

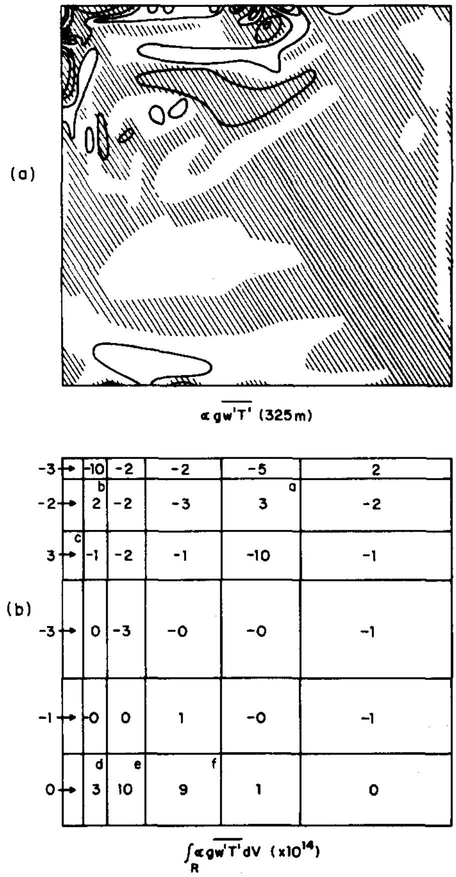


Fig. 6. Mean eddy buoyancy work. (a) In the upper thermocline (325 m), as Fig. 2a. Positive value indicates conversion of mean potential energy into MKEF. (b) Subregional integrals as in Fig. 3b.

is generally representative of the other levels. Regions in which the eddy buoyancy work is negative, indicating conversion of MKEF into potential energy, are indicated by stippling. This is again a log-contour plot with the first contour line at 10^{-5} erg g^{-1} s^{-1}, the next at 10^{-4}, etc. The largest values are found in the northwestern part of the WBC and along the boundary in the NBC(E), where point values can be 10^{-3} erg g^{-1} s^{-1}, but there is considerable grid-scale variation. The largest area of large point values and a single sign is the SW, where maximum values are O (10^{-5}). There are also relatively large regions of significant positive and negative values in the NBC(W) and Near Field, respectively.

The subregional integrals (Fig. 6b) reveal that only in the SW, in the part of the NBC(W) where it is turning from southward flow to westward flow, in two small subregions of the WBC and in a narrow region in the northeast, is there positive net eddy buoyancy work. The areas of largest point values generally have integrated values smaller than those of the SW subregions.

The vertical distribution of eddy buoyancy work has been examined in the labeled subregions of Fig. 6b, à la Table II. Because the values are not generally large enough to play a major role in most balances, only the qualitative behavior is described. The largest eddy buoyancy work integrals are generally found in the upper thermocline (160—490 m) or thermocline (490—1310 m) depth bands in the northern and western subregions. In the southwestern subregions the deep (1310—2690 m) contribution is comparable to the thermocline and upper thermocline contributions. Examination of the volume normalized forms of these integrals reveals that the largest value is generally in the upper thermocline band. There can be more than an order of magnitude decrease of normalized values within the water column in the northern and western subregions, and typical maximum values are O (10^{-6}) erg cm^{-3} s^{-1}.

3. *Summary*

Interaction work terms are systematically large in some parts of the NBC-(W) and/or Near Field regions. The vertically integrated fields can conveniently be used in these regions because there is little or no sign variation with depth. There is loss of KEM in the eastern and central parts of the NBC(W) and gain of MKEF in the central and eastern parts of the NBC(W) and Near Field. Generally the subregions of the NBC(W) and Near Field are not of Type 1 and so it is inappropriate to identify any single subregion as a primary "source" region because interaction work does not generally act as a conversion process locally within these regions (see Section 5). However, integrated over the horizontal extent of the basin within individual depth bands, interaction work does act as a conversion process, and converts KEM to MKEF over each depth interval.

There is considerable spatial structure in the eddy buoyancy work term, but subregional integration generally leads, even in areas of maximum point values, to integrated values which are smaller than the corresponding regional MKEF interaction work values. Only in the SW subregions is buoyancy work consistently larger than MKEF interaction work.

B. Horizontal and vertical diffusion work

The only significant dissipation or diffusion of MKEF due to vertical diffusive processes occurs through bottom stress work in the deep water. The largest subregional vertical diffusion work in the upper four levels is less than 10^{13} erg s^{-1} and would be entered as zero on the scale of the previous figures. The distribution of bottom stress dissipation is as one would expect — greatest in the NBC(W) and Near Field regions, where the eddy signal is strongest, and reasonably uniform with reduced magnitude elsewhere. Fig. 7a presents the subregional integrals of eddy vertical diffusion work.

Given the highly barotropic character of the Interior eddy field of (RHMS), the pointwise distribution of horizontal diffusion work is predictably uniform with depth. Even in the intense current regions it varies at maximum by a factor of five from the surface level to the deepest level. Fig. 7b presents the subregional integrals of eddy horizontal diffusion work and reveals that near the boundaries, integrated horizontal diffusion work tends to be larger than vertical diffusion work, but elsewhere they are comparable in magnitude.

C. Boundary transport processes

1. Mean flow transport

The term that describes the divergence of the mean flow MKEF flux vector, $\partial\,[\overline{u_j(\frac{1}{2}u'_\lambda u'_\lambda)}]/\partial x_j$ is shown, vertically integrated, in Fig. 8a. The largest point values are found along the northern boundary in the separation region, in the NBC(W) at the longitude of boundary current separation, and in the middle of the NBC(W)/Near Field region west of the separation longitude. There is mean flow scale variation in the NBC(W)/Near Field where the divergence is systematically positive in the eastern part of the NBC(W) and generally negative elsewhere.

Subregional integrals are shown in Fig. 8b, and confirm the above. Recall that:

$$\int_R \frac{\partial}{\partial x_j}\phi_j\,\mathrm{d}V = \int_{S_R} \phi_j n_j\,\mathrm{d}S$$

where ϕ_j is a vector, S_R is the surface bounding the region R and n_j is the unit outward normal vector to the element of surface dS. Thus these integrals give the rate at which MKEF is leaving the region due to mean transport. A positive value implies that MKEF is leaving the subregion. There is strong export of MKEF from the eastern subregion of the NBC(W) and comparably strong import in the subregions west and south of it. Nowhere else in the basin is there strong net import or export of MKEF by the mean flow.

(a)

-0 →	-0	-0	-0	-0	-0
-0 →	-0	-0	-5	-16	-2
-0 →	-0	-0	-15	-26	-4
-0 →	-0	-1	-4	-8	-10
-0 →	-0	-1	-4	-5	-7
-0 →	-0	-2	-5	-3	-7

$\int_R k u'_\lambda \overline{\frac{\partial^2 u'_\lambda}{\partial z^2}}\, dV$ $(\times 10^{14})$

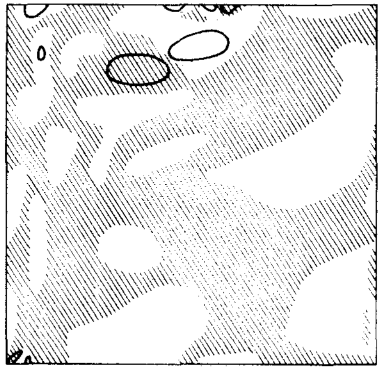
(b)

-1 →	-1	-1	-1	-5	-1
-5 →	-2	-2	-10	-16	-2
-3 →	-3	-2	-12	-14	-2
-1 →	-2	-2	-2	-4	-7
-0 →	-0	-1	-4	-4	-5
-0 →	-2	-4	-6	-2	-5

$\int_R A_m u'_\lambda \overline{\frac{\partial^2 u'_\lambda}{\partial x_\beta \partial x_\beta}}\, dV$ $(\times 10^{14})$

Fig. 7. Mean eddy diffusion work subregional integrals. (a) Vertical diffusion work. (b) Horizontal diffusion work. Negative values indicate decrease in subregional MKEF. Units are 10^{14} erg s^{-1}.

(a)



$\int_{-H}^{0} \frac{\partial}{\partial x_j}\left[\bar{u}_j\left(\tfrac{1}{2}\overline{u'_\lambda u'_\lambda}\right)\right] dz$

(b)

1 →	10	-4	1	7	1
-1 →	4	-6	-56	93	-3
0 →	1	-8	-9	-12	-3
0 →	-2	3	-4	-3	0
-1 →	0	-1	-0	-1	-1
-5 →	2	-2	2	0	-1

$\int_R \frac{\partial}{\partial x_j}\left[\bar{u}_j\left(\tfrac{1}{2}\overline{u'_\lambda u'_\lambda}\right)\right] dV$ $(\times 10^{14})$

Fig. 8. Mean flow transport of MKEF. (a) Vertically integrated term, (b) Subregional integrals, conventions as in Fig. 3. For this, and all other transport processes, negative values indicate an influx of MKEF.

2. *Mean eddy flow transport*

Consider the divergence of the mean eddy advection of eddy energy transport vector, $\partial(\tfrac{1}{2}\overline{u'_j u'_\lambda u'_\lambda})/\partial x_j$. The statistical reliability of the point values of this term is the poorest of the terms considered. The field is not statistically significantly different from zero at the 95% level except in the NBC(W) and Near Field regions, where the largest point values have error bars equal to about half their value. To establish significance in the Interior values would require a time series at least ten times longer than the analysis record used in (RHMS).

Fig. 9 shows a plot of the vertically integrated point values and the subregional integrals of this field. Only in the NBC(W) and Near Field do the subregional integrals attain any non-trivial magnitude. Furthermore, the net transfer of MKEF out of the combined NBC(W)/Near Field region is negligible. Within the NBC(W)/Near Field this flux is comparable to or smaller than mean flow flux.

3. Mean eddy pressure work transport

Now consider the divergence of the eddy pressure work, $\partial\, \overline{u_j' \tilde{p}'}/\partial x_j$. This term has been evaluated as a residual from all of the other terms in the MKEF equation, because it does not appear naturally in the energy equation in a form that can be computed easily from the momentum equation finite difference expressions. In order to verify the magnitudes and distribution of the divergence of the eddy pressure work, a standard second order centered difference approximation to $\partial\, \overline{u_j' \tilde{p}'}/\partial x_j$ was computed and integrals of this compared with those of the residual term. They compare satisfactorily in sign and magnitude, and so no significant error should be introduced by use of the residual term.

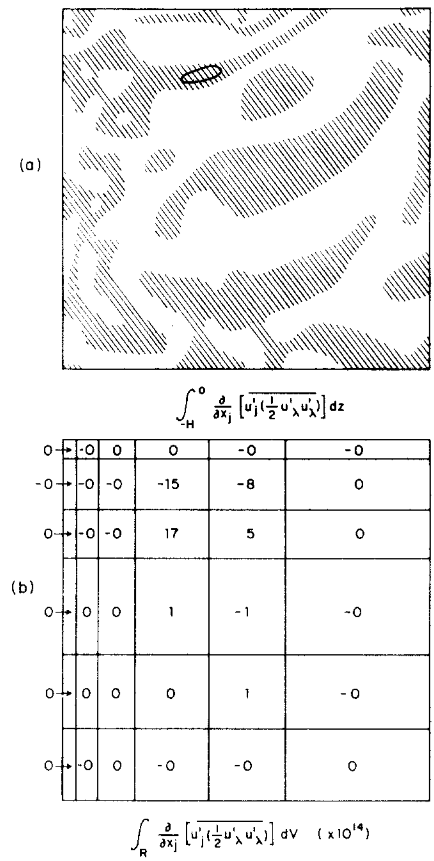
Fig. 10a shows a plot of the vertically integrated point values. As before, there is some very small scale spatial structure in the northwestern part of the basin and on the northern boundary where the NBC(E) separates from the wall, but elsewhere the term is generally smoothly distributed. Throughout the Interior the horizontal divergence is negative, indicating import of MKEF by eddy pressure work. There are areas of significant positive and negative values in the NBC(W) and Near Field regions.

4. Summary

Figs. 8b, 9b and 10b should be compared in order to determine the relative importance of the different transport processes. The Interior subregions show eddy pressure work influx at typically 5 to 10 times the rate of mean flow influx and mean eddy flow transport is negligible. There is no systematic behavior in other regions, particularly in the NBC(W) and Near Field where each transport term can be important and adjacent subregions can have opposite signs for a particular process.

A convenient way to illustrate the relative importance of these transport terms is to examine the net southward transport of MKEF due to each process as a function of latitude. Since there can be no transport through the boundaries of the domain, it is possible to do this calculation by appropriately summing the subregional integrals starting at, say, the southern boundary and working northward in latitude bands. By this summation process it is easy to evaluate:

$$\int_{-H}^{0} \int_{0^\circ \mathrm{W}}^{22^\circ \mathrm{W}} \int_{26^\circ \mathrm{N}}^{\theta^\circ \mathrm{N}} \frac{\partial}{\partial x_j} (\phi_j)\, \mathrm{d}V = \int_{-H}^{0} \int_{0^\circ \mathrm{W}}^{22^\circ \mathrm{W}} (\phi_y|_{\theta^\circ \mathrm{N}})\, \mathrm{d}V$$



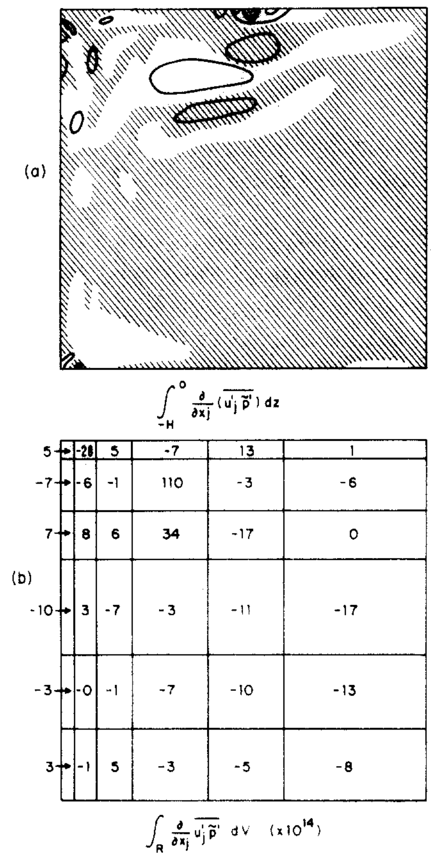


Fig. 9. Mean eddy flow transport of eddy kinetic energy. As in Fig. 8.

Fig. 10. Mean eddy pressure work transport of MKEF. As in Fig. 8.

where ϕ_j is one of the transport process vectors, ϕ_y is its meridional component, and the model boundary conditions, together with the divergence theorem, are used to obtain the right hand integral.

The results of this calculation are shown in Fig. 11. Within the NBC(W) and Near Field the three transport terms all contribute at $O(1)$ to the meridional transport of MKEF. However, only the pressure work term is responsible for any substantial southward transport of MKEF south of the Near Field southern boundary latitude. It should be recalled that each of the transport terms can be O (1) locally within the NBC(W) and Near Field as well as in the net. There is important redistribution of MKEF by boundary transports within these regions.

A similar calculation to obtain the net eastward transport of MKEF reveals that the pressure work term accounts for 80% of the net eastward transport of

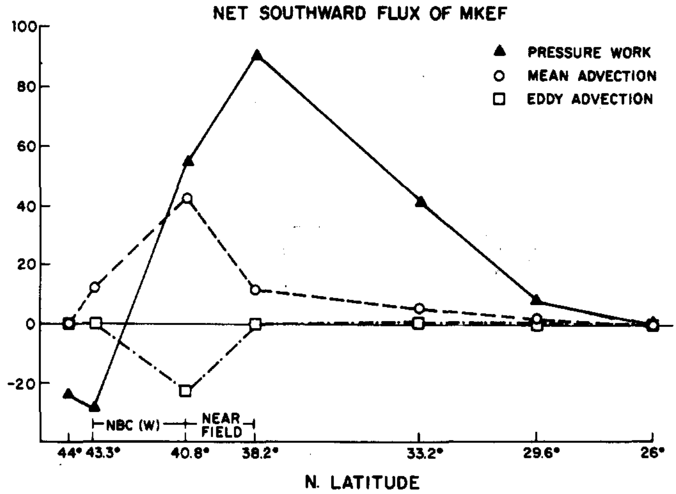


Fig. 11. Net southward flux of MKEF, as a function of latitude, for each of the boundary transport processes. Note that within the NBC(W) and Near Field region latitudes all transport processes are of the same order of magnitude, but that further south pressure work becomes the dominant transport process. Units are 10^{14} erg s^{-1}. (See text.)

the energy into the part of the basin east of the eastern boundary of the NBC regions.

4. REGIONAL MKEF BUDGETS

From the preceding section it is clear that the spatial characteristics and relative magnitude of different energetic processes can be complex. Furthermore, in the NBC(W) and Near Field areas of the domain it is possible to find subregions over which each of the terms of Section 2. C is O (1) compared to the largest term, yet the change due to a particular process can change sign between adjacent subregions. Examined on the scale of Section 3, the local MKEF energetics do not provide a satisfactory overall description of the energetic processes responsible for the features of this flow. In this section energy budgets over larger regions are examined in the spirit of Section 2. D. Budgets over the dynamically distinct regions of RHMS are first described and then followed by budgets over some useful alternative regions.

The terms of Section 2. C are presented in the schematic form of Fig. 12a. The MKEF Reynolds stress—mean flow interaction work term is at the top of the box; mean eddy horizontal and vertical diffusion work are on the right side; mean eddy buoyancy work is at the bottom; mean flow boundary MKEF transport, mean eddy flow boundary transport, and mean eddy pressure work boundary transport are on the left. The sign of the energy change in the region due to a particular process is indicated by the direction of the arrow and

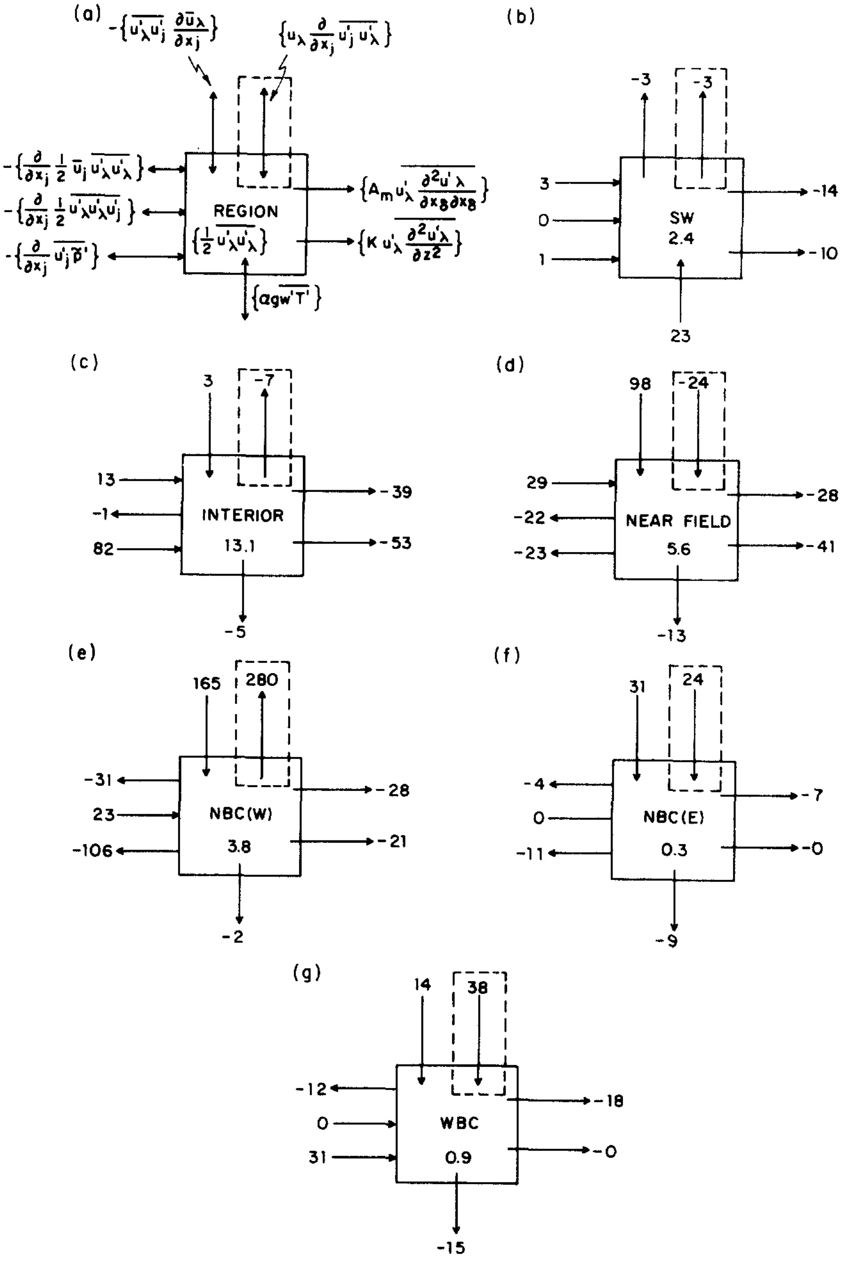


Fig. 12. RHMS kinematical and dynamical regional MKEF budgets (Fig. 2a). See text for description. Regional MKEF units are 10^{22} erg, while regional work units are 10^{14} erg s^{-1}

sign of the value, with an arrow pointing out of the box corresponding to a decrease in regional MKEF and a negative value.

The term at the top of the box in the dashed region is the KEM Reynolds stress—mean flow interaction work term. The sign convention adopted for display of the KEM interaction work term is that an arrow into the box (positive value) corresponds to a decrease of regional KEM due to interaction work. Thus a Type 1 region is identified by having the magnitude and direction of both interaction work arrows the same.

A. *RHMS regional budgets*

The RHMS regional budgets are presented in Fig. 12b—g and the dominant balance region-by-region is:

1. Southwest (Fig. 14b): Eddy buoyancy work conversion of potential energy is balanced by horizontal and vertical diffusion work losses. Region is of Type 1.

2. Interior (Fig. 14c): Eddy pressure work influx is roughly balanced by horizontal and vertical diffusion work losses. Region is of Type 2.

3. Near Field (Fig. 14d): All processes enter the lowest balance. MKEF interaction work and mean flow transport increase MKEF while other processes provide balancing losses. KEM and MKEF are both increased by interaction work. Region is of Type 3.

4. NBC(W) (Fig. 14e): All processes save buoyancy work enter the lowest order balance. MKEF interaction work and mean eddy transport increase MKEF while the other processes decrease it. MKEF interaction work and eddy pressure work transport are the largest terms. KEM is decreased at twice the rate that MKEF is increased by interaction work. Region is of Type 3.

5. NBC(E) (Fig. 14f): MKEF interaction work increases MKEF and is balanced by eddy pressure work transport, eddy buoyancy work, horizontal diffusion work and mean flow transport losses. Region is roughly of Type 1.

6. WBC (Fig. 14g): Eddy pressure work transport and MKEF interaction work increase MKEF while horizontal diffusion work, eddy buoyancy work and mean flow transport provide the balance. KEM is decreased at three times the rate that MKEF is increased. Region is of Type 3.

In none of these regions is the basin integrated MKEF balance — Reynolds stress—mean flow interaction work conversion of KEM into MKEF balanced by horizontal and vertical diffusion work losses — found to exist. A typical time for the replacement of the MKEF in a region due to the sum of all processes which increase MKEF is $\sim$20 days in intense current regions and 50—150 days elsewhere. The time for diffusion work to remove the regional MKEF assuming diffusion work continues at the given level is $\sim$50 days in intense current regions and 100—150 days elsewhere.

The Interior and SW regional budgets are easily interpreted, because these regions are of Type 2 and Type 1, respectively and are simple, involving a limited number of processes. The other regional budgets involve considerably more processes and the regions are generally not of Type 1 or 2. Especially in the NBC(W) and Near Field the number of processes in the dominant balance, the Type 3 character of the region and the fact that MKEF interaction work is the largest term combine to limit the possibility of simple physical interpretation of these energetically important regions.

B. Budgets over alternative regions

Here MKEF budgets over alternative regions are examined to try to increase understanding of the energetic processes in the areas of the basin where the budgets of Section 4. A are not wholly satisfactory. First the effects of combining together the NBC(W) and Near Field as well as the NBC(E) and WBC are described and then the WBC, NBC(E) and NBC(W) are slightly redefined and new budgets presented.

Fig. 13 presents MKEF budgets for the combined NBC(W)/Near Field and WBC/NBC(E) regions. The combined NBC(W)/Near Field/MKEF balance is

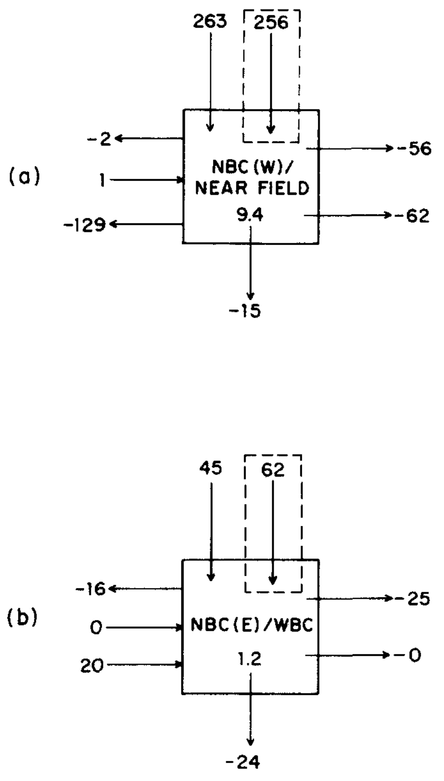


Fig. 13. MKEF budget for the combined NBC(W)/Near Field and NBC(E)/WBC regions. See text for description. Units as in Fig. 12.

quite simple (Fig. 13a) because the combined region is of Type 1 and important transport effects are fewer. There is large input of MKEF due to Reynolds stress—mean flow interaction work conversion of KEM into MKEF which is roughly balanced by pressure work transport and diffusion work losses. The combined WBC/NBC(E) regions are almost a Type 1 region and show a budget in which there is little net transport. Input of MKEF due to interaction work is nearly balanced by losses due to horizontal diffusion work and eddy buoyancy work.

Although there would seem to be a compelling mean flow kinematical argument for separating the NBC(W) and Near Field regions when the mean flow is examined (Fig. 1), consideration of maps of MKEF and of the mean and instantaneous flow balance of terms (RHMS Sections 3. D.2 and 4) reveals that their combination can be defended on eddy flow kinematical and dynamical grounds. In addition, since interaction work represents the dominant energetic process in each of the regions, the increased ease of interpretation accorded by the fact that the combined region is Type 1 further supports their combination.

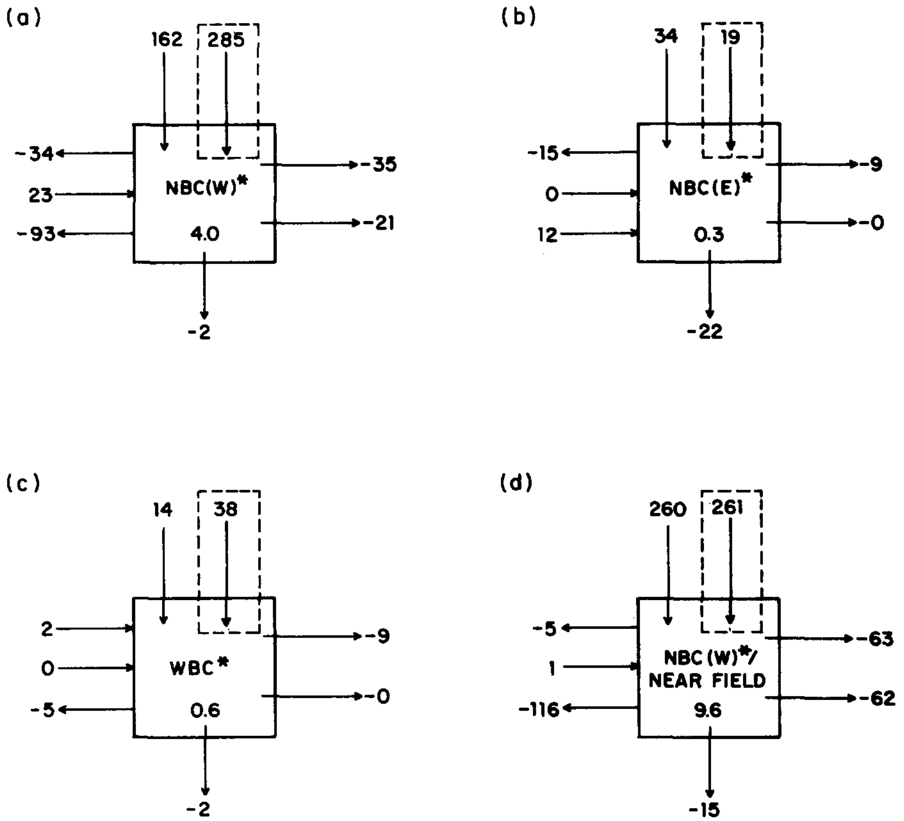


Fig. 14. Alternate kinematical regional MKEF budgets (Fig. 1b), as in Fig. 12.

The best choice of regions for the western and northern boundary currents is less clear. A kinematically defensible alternative to the NBC(E) and WBC involves the redefinition shown in Fig. 1b. This selection of regions emphasizes the recirculation aspect of the northern current system, in which over two thirds of the mass transport never leaves the NBC(E) * and NBC(W) * regions, and in which the WBC * region simply includes the net northward transport required to balance the Interior and SW transports. The energetics of these regions and of the combined NBC(W) */Near Field region are shown in Fig. 14.

Using these new regions the NBC(W) * and NBC(W) */Near Field regional budgets are not modified in any significant way from their counterparts but the WBC * and NBC(E) * budgets are importantly different from those of the WBC and NBC(E). Within the WBC * input of energy due to MKEF interaction work is roughly balanced by diffusion work while within the NBC(E) * the MKEF interaction work input is roughly balanced by losses to eddy buoyancy work and horizontal diffusion work. There is little net export or import of MKEF in either Type 3 region.

5. DISCUSSION

A. *Summary of RHMS energetics*

The set of regions consisting of the NBC(W)/Near Field (or NBC(W) */ Near Field), Interior and SW account for 95% of the basin MKEF and suffice to permit a physically satisfactory and simple description of the MKEF energetics over 90% of the basin volume. Within the NBC(W)/Near Field KEM is converted to MKEF via Reynolds stress—mean flow interaction work, about half is lost to diffusive processes and most of the rest is exported bia eddy pressure work. The eddy field in the Interior is maintained against dissipation predominantly by the influx of energy from eddy pressure work; direct conversion from other forms of energy and influx by other boundary transports are small. The eddy field in the SW is maintained predominantly by eddy buoyancy work conversion of potential energy into MKEF; other inputs are negligible.

Within the narrow boundary current regions of the WBC and NBC(E) the MKEF balances are more complicated. These were seen in (RHMS) to be the most dynamically complex regions of the mean flow, with one grid interval wall boundary layers, considerable variation in dynamical balances within each region and some suggestion of erratic numerical behavior in the extreme northwest corner and in the separation region of the NBC(E). It is not surprising, therefore, that the local energetic properties of these regions seem to be less than simple. However, the results of Section 4. B illustrate that re-definition of the analysis regions leads to the conclusion that the regions are energetically passive insofar as serving as source regions or as sink regions of MKEF produced elsewhere.

The spatial inhomogeneity of the model flow kinematics and dynamics has been seen to have a counterpart in similarly inhomogeneous regional MKEF balances. The regions identified in RHMS through kinematical and balance of terms analysis can be used to usefully describe the most important features of the MKEF energetics over most of the basin, so long as the NBC(W) and Near Field are treated as a single region. The energetics of the principal source region are simple only when the whole region is considered as a unit. Attempts to form useful budgets over subregions of the larger region have proven fruitless due to the large number of processes at work and to interaction work interpretation difficulties.

B. Interaction work interpretation and conversion

Determination of the NBC(W)/Near Field as the primary source region of MKEF for the basin resulted from examination of both the KEM and MKEF interaction terms and their subregional integrals. It is not possible to define any smaller region as a primary conversion region.

The energetic interpretation of this source region would be significantly altered if the interaction work interpretations used by Holland and Lin (1975) or by Semtner and Mintz (1977) were employed. Semtner and Mintz interpret the KEM interaction term as a conversion term between KEM and MKEF ("the barotropic generation term", pp. 226—227 and fig. 19) while Holland and Lin similarly interpret a two-layer analog of the MKEF interaction term ("the conversion of mean kinetic to eddy kinetic energy by the Reynolds stresses," p. 649 and fig. 10). Using the Semtner and Mintz interpretation we would conclude from Fig. 4 that MKEF results almost entirely from conversion of KEM in the eastern part of the NBC(W) region. The importance of the Near Field region would be overlooked. The Holland and Lin interpretation properly describes where MKEF is being increased by interaction processes, but leads one to conclude from Fig. 3 that the mean flow is losing energy to the eddies throughout the NBC(W)/Near Field.

The KEM interaction term describes only the changes in KEM, just as the MKEF interaction term describes only changes in MKEF. Knowledge of one term generally does not give any information about the other, because of the lack of a pointwise conversion property for interaction work (Section 2. B). This same property means that ideas of "positive or negative eddy viscosity" as used by Semtner and Mintz (p. 227) and Holland and Lin (pp. 649, 654), by which is understood conversion of KEM to MKEF or vice versa, are pointwise inappropriate in general. Only when the physical significance of the KEM and MKEF interaction terms is properly understood is the complexity of the interaction process correctly described.

C. Remarks

Vertically integrated regional energy balances do not generally tell the entire local energetics story for a flow, but in this experiment it has been shown by the results of Section 3 that vertical integration involves no essential loss of information, especially in the primary source region. In particular integration of the interaction work terms over the NBC(W)/Near Field and the horizontal area of the basin, and for each depth band reveals that interaction works acts to convert KEM to MKEF over every one of these regions. Thus the eddies do not directly drive a mean flow in the large or in the source region at any depth, but draw upon the mean flow for their energy. Although the result is far from definitive at this time, Schmitz (1977) has evidence to suggest that the deep oceanic Gulf Stream near field may have the opposite direction of energy flow.

The source of vertical transfer of KEM beneath the wind-driven surface level, which is necessary if the flow is to be in equilibrium and also provide energy for the eddy field at each level, appears to be mean vertical pressure work transport from the wind-driven upper level to the deeper levels.

This type of regional energy analysis is a useful tool for dynamical investigation of numerical model flows and, in principle, of oceanic flows. Its utility is constrained only by the need for the many different energy terms. For numerical model analysis less effort is required to evaluate these terms than might be expected; this analysis and that described in RHMS required less than 1% of the computer time required to do the integration of the basic experiment of (RHMS). Regional energy analysis makes possible, with relatively little effort, physically meaningful differentiation between different model flows and provides useful information about the physical processes responsible for the flow in its distinct regions.

In addition to being a useful tool for the study of numerical ocean experiments, the analysis ideas used here are directly applicable to interpretation of the increasing amounts of time averaged current meter data being obtained through various oceanographic field programs (e.g., MODE, POLYMODE) or of Reynolds averaged atmospheric data. The problems of inferring dynamics from data apply equally well to laboratory, numerical model and oceanographic field data sets. An understanding of the interpretation limits inherent in a given data set is important for proper experimental design of oceanographic programs.

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This is MODE contribution No. 101 (POLYMODE).

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