Stochastic parameterization for deep ocean convection

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Preliminaries on convection

Why deep ocean convection matters in climate?

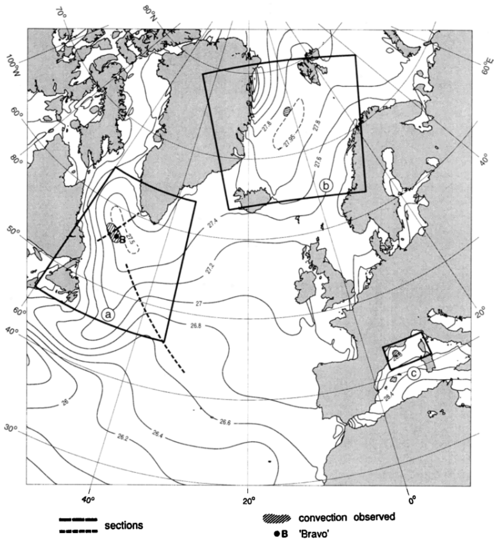


Figure 1: Sites of deep convection in the North Atlantic (Marshall and Schott, 1999)

- Major deep ocean convection sites are located in the North Atlantic ocean:
  - Labrador Sea
  - Greenland Sea
  - Western Mediterranean
- Control deep water formation rate.
- Communicates surface atmospheric properties (e.g. heat, carbon) to the abyssal ocean.

Why deep ocean convection matters in climate?

- North Atlantic deep convection in current climate models (CMIP6) is too **deep**, too **frequent**, and too **broad**.

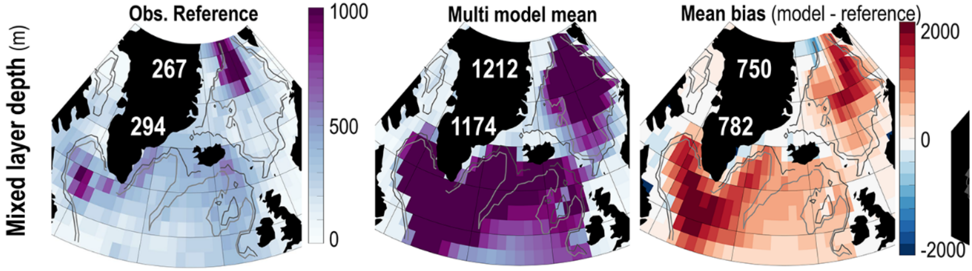


Figure 2: CMIP6 deep ocean convection biases in the North Atlantic (Heuzé, 2021).

→ Expected to worsen in future climate models with increased ocean resolution (Heuzé, 2021; Fox-Kemper et al., 2021).

Climate models rely on Hydrostatic ($\delta = \frac{\mathcal{H}}{\mathcal{L}} \ll 1$) Primitive Equations (HPE) ocean models.

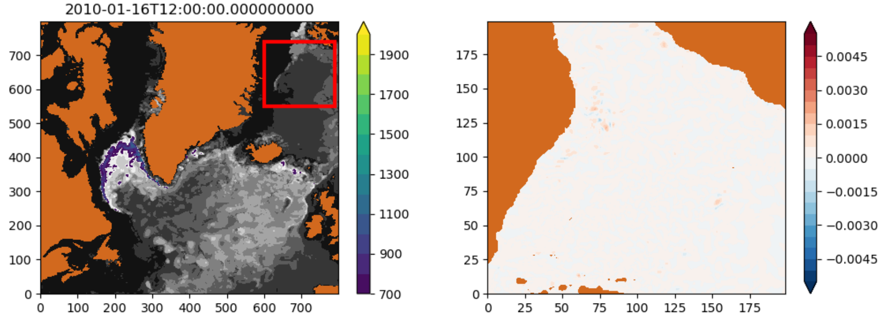


Figure 3: eORCA12 North Atlantic deep convection event.

Climate models rely on Hydrostatic ($\delta = \frac{\mathcal{H}}{\mathcal{L}} \ll 1$) Primitive Equations (HPE) ocean models

$$\partial_t u + \boldsymbol{\nabla}.(\boldsymbol{u}u) - fv - \tilde{f}w = -\frac{1}{\rho_0}\partial_x p + \mathcal{F}_u + \mathcal{D}_u, \tag{1a}$$

$$\partial_t v + \boldsymbol{\nabla}.(\boldsymbol{u}v) + fu = -\frac{1}{\rho_0}\partial_y p + \mathcal{F}_v + \mathcal{D}_v, \tag{1b}$$

$$\partial_t w + \boldsymbol{\nabla}.(\boldsymbol{u}w) - \tilde{f}u = -\frac{1}{\rho_0}\partial_z p - \frac{\rho}{\rho_0}g + \mathcal{F}_w + \mathcal{D}_w, \tag{1c}$$

Note: **black** → Hydrostatic ; blue → Non-Hydrostatic + non-traditional Coriolis

Q1: How to re-introduce vertical dynamics, but avoiding a full Non-Hydrostatic model (Klingbeil and Burchard, 2013; Garreau, 2021)?

Q2: Can stochastic modelling help in the process?

Our approximations/hypotheses:

- Non-traditional (horizontal) Coriolis terms are neglected.
- Vertical acceleration remains diagnostic (through continuity):

$$(\mathrm{d}_t + \boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} = (\mathrm{d}_t + \boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u}_h \tag{2}$$

- High resolution/strong noise regime, with aspect ratio $\delta = \frac{\mathcal{H}}{\mathcal{L}} \sim \mathcal{O}(1)$.

The stochastic transport equation for the vertical velocities reads:

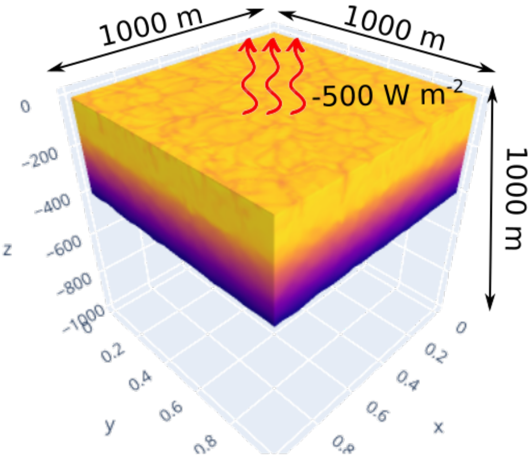
$$(-\frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{a}) \cdot \boldsymbol{\nabla} w \mathrm{d}t - \frac{1}{2}\nabla \cdot (\boldsymbol{a}\boldsymbol{\nabla} w)\mathrm{d}t + \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t \cdot \boldsymbol{\nabla} w = (-\frac{1}{\rho_0}\partial_z p + b)\mathrm{d}t - \frac{1}{\rho_0}\partial_z \mathrm{d}p_t^\sigma, \tag{3}$$

Pressure field + NH correction is obtained through vertical integration

$$\mathrm{d}p_t^\sigma(z) = \mathrm{d}p_t^\sigma|_{z=\eta} - \rho_0 \int_z^\eta (\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t \cdot \boldsymbol{\nabla} w)\, dz' \tag{4}$$

$$p(z) = p|_{z=\eta} + \rho_0 g(\eta - z) + \rho_0 \int_z^\eta \left(b - \underbrace{\left((\frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{a}) \cdot \boldsymbol{\nabla} w + \frac{1}{2}\nabla \cdot (\boldsymbol{a}\boldsymbol{\nabla} w) \right)}_{b_{NH}} \right) dz' \tag{5}$$

Idealized Large Eddy Simulation (LES) of free convection.



- **Physics**:
  - $N^2 = 2 \times 10^{-6}$ s^{-2}
  - $Q_{net} = -500$ W m^{-2}
  - 3 days long run
  - Linear EOS: $\rho = \rho_0(1 - \alpha_T T)$
- **Numerics**:
  - 1 km x 1 km x 1 km
  - Isotropic 10 m resolution
  - doubly periodic
- **Three experiments**:

NH Non-hydrostatic, non-Boussinesq (NBQ, Auclaire et al., 2018)

Hydro Hydrostatic, KPP vertical closure

Q-NH Quasi-NonHydrostatic, KPP vertical closure, Laplacian dissipation on w with $\nu = 1$m^{2}s^{-1}

Preliminary implementation in ROMS-CROCO:

- Baroclinic pressure correction:

$$p(z) = p|_{z=\eta} + \rho_0 \int_z^\eta (b - b_{NH}) \; dz' \tag{6}$$

with

$$b_{NH} = \sum w\_trends. \tag{7}$$

- Baroclinic-barotropic coupling term:

$$\overline{\rho}_{NH} = \frac{1}{H} \int_{-H}^{\eta} \rho_{NH} \; dz' \quad ; \quad \rho^*_{NH} = \frac{1}{\frac{1}{2}H^2} \int_{-H}^{\eta} P_{NH} \; dz' \tag{8}$$

- Pressure-correction only applied at *corrector* time-step.
- Noise = horizontal Laplacian with constant coefficient

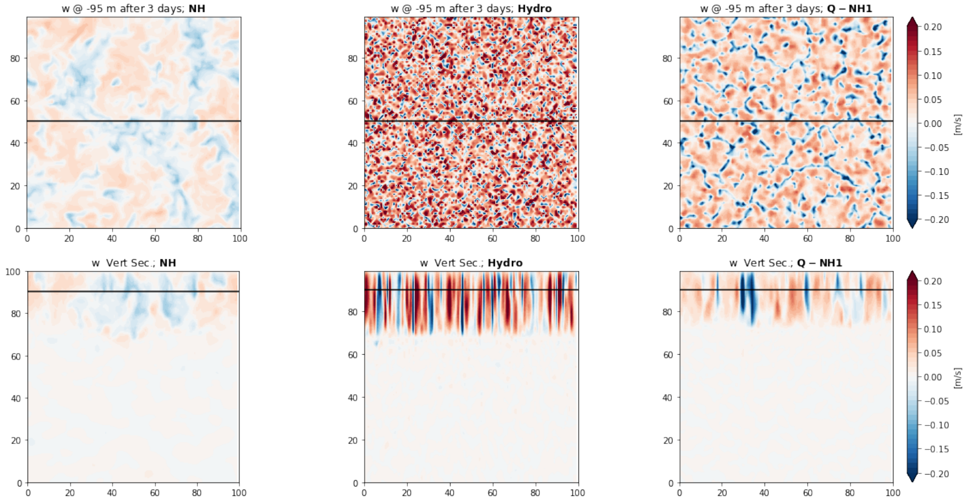


Figure 4: Horizontal (top) and vertical (bottom) sections after 3 days of simulation for the **NH** (left) and the **Hydro** (center) and **Q-NH** (right) experiment

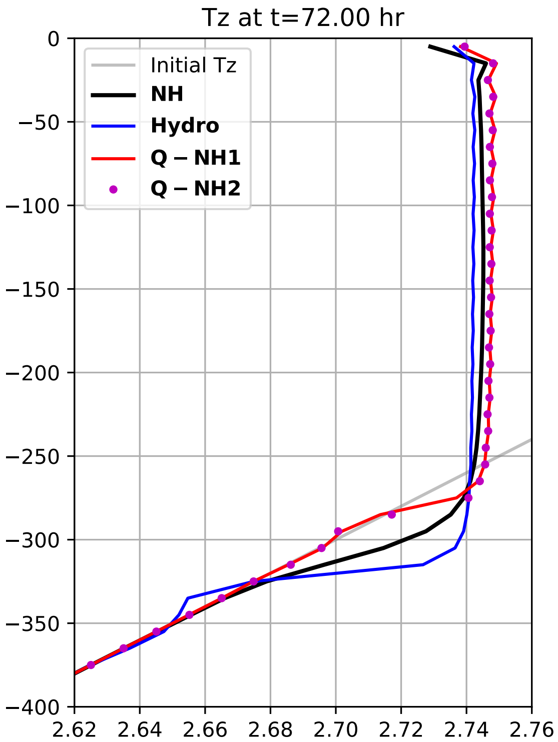


Figure 5: Vertical profiles of horizontally averaged temperature for the different experiments. **Q-NH1** uses Laplacian viscosity and **Q-NH2** Smagorinsky-like formulation.

- Deep ocean convection is crucial for large scale ocean circulation, yet it is poorly represented in climate models.
- Direct nonhydrostatic pressure correction approach offers an attractive avenue for preliminary tests of stochastic convective parameterization.

- Implement stochastic advection associated with Laplacian noise, i.e.

$$\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t \cdot \boldsymbol{\nabla}\, w = \sum_k \gamma_k \lambda_k e_k(\mathbf{x}) \tag{9}$$

  with

$$\lambda_k = \left\langle \sqrt{2\epsilon dt} |\nu^{1/2} \boldsymbol{\nabla} w|; e_k \right\rangle_{\mathcal{L}^2} \tag{10}$$

- Moving from $\delta \sim \mathcal{O}(1)$ to $\delta \ll 1$, i.e. toward climate model applications.